On the Regularity of the Extremal Controls of a Class of Optimal Control Problems

Martin Ivanov[1], Mikhail Krastanov[1,2], and Nadezhda Ribarska[1,2]

[1]Sofia University "St. Kliment Ohridski", Sofia, Bulgaria
[2]Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, Sofia, Bulgaria

Abstract

In the early 1960s, Fuller [2] proposed a control problem in which each optimal trajectory has countably many switching points that accumulate to the terminal point. This phenomenon, i.e. the existence of infinitely many switching times on a finite time interval, is called chattering. Later, in the mid-1980s, Kupka [3] showed that chattering is structurally stable in spaces of sufficiently high dimension. It is natural to ask whether the countably many switchings with a finite number of accumulation points, observed in the Fuller problem, is the worst form of chattering that can occur.

The answer to this question is negative for the class of C^∞ control-affine problems: Sussmann [4] proved that in this case no irregularity of the optimal controls can be excluded a priori. Yet, in a generic case (which does not include the Fuller problem), Boarotto and Sigalotti [1] obtained a bound on the order of iterated accumulations of switchings. For the class of real-analytic control-affine problems, the stated question is still open and only partial results [4] are known.

Here, we give a positive answer to the stated question question for a class of control-affine problems that includes the Fuller problem. More precisely, we consider the control system

$$\dot{x}(t) = f(x(t)) + u(t)g(x(t)), \quad u(t) \in [-1, 1], \tag{1}$$

where f and g are sufficiently smooth vector fields on $\mathbb{R}^n$ and the controls are Lebesgue measurable functions with values in $[-1, 1]$. Under conditions that guarantee the occurrence of chattering, we prove that each extremal trajectory (that is, each trajectory of (1) which satisfies the Pontryagin Maximum Principle for boundary trajectories associated with (1)) is smooth except for at most countably many points. The set of times at which an extremal trajectory is not smooth consists of the switching times and their accumulation points, which are proved to be at most finitely many.

References

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